HW3CSE105F24: Homework assignment 3

CSE105F24

Due: October 22nd at 5pm, via Gradescope

In this assignment,

You will demonstrate the richness of the class of regular languages, as well as its boundaries.

Resources: To review the topics for this assignment, see the class material from Week 3. We will post frequently asked questions and our answers to them in a pinned Piazza post.

Reading and extra practice problems: Sipser Chapter 1. Chapter 1 exercises 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 1.10, 1.11, 1.12, 1.14, 1.15, 1.16, 1.17, 1.19, 1.20, 1.21, 1.22. Chapter 1 problem 1.51.

For all HW assignments: Weekly homework may be done individually or in groups of up to 3 students. You may switch HW partners for different HW assignments. Please ensure your name(s) and PID(s) are clearly visible on the first page of your homework submission and then upload the PDF to Gradescope. If working in a group, submit only one submission per group: one partner uploads the submission through their Gradescope account and then adds the other group member(s) to the Gradescope submission by selecting their name(s) in the "Add Group Members" dialog box. You will need to re-add your group member(s) every time you resubmit a new version of your assignment. Each homework question will be graded either for correctness (including clear and precise explanations and justifications of all answers) or fair effort completeness. For "graded for correctness" questions: collaboration is allowed only with CSE 105 students in your group; if your group has questions about a problem, you may ask in drop-in help hours or post a private post (visible only to the Instructors) on Piazza. For "graded for completeness" questions: collaboration is allowed with any CSE 105 students this quarter; if your group has questions about a problem, you may ask in drop-in help hours or post a public post on Piazza.

All submitted homework for this class must be typed. You can use a word processing editor if you like (Microsoft Word, Open Office, Notepad, Vim, Google Docs, etc.) but you might find it useful to take this opportunity to learn LaTeX. LaTeX is a markup language used widely in computer science and mathematics. The homework assignments are typed using LaTeX and you can use the source files as templates for typesetting your solutions. To generate state diagrams of machines,you can (1) use the LaTex tikzpicture environment (see templates in the class notes), or (2)) use the software tools Flap.js or JFLAP described in the class syllabus (and include a screenshot in your PDF), or (3) you can carefully and clearly hand-draw the diagram and take a picture and include it in your PDF. We recommend that you submit early drafts to Gradescope so that in case of any technical difficulties, at least some of your work is present. You may update your submission as many times as you'd like up to the deadline.

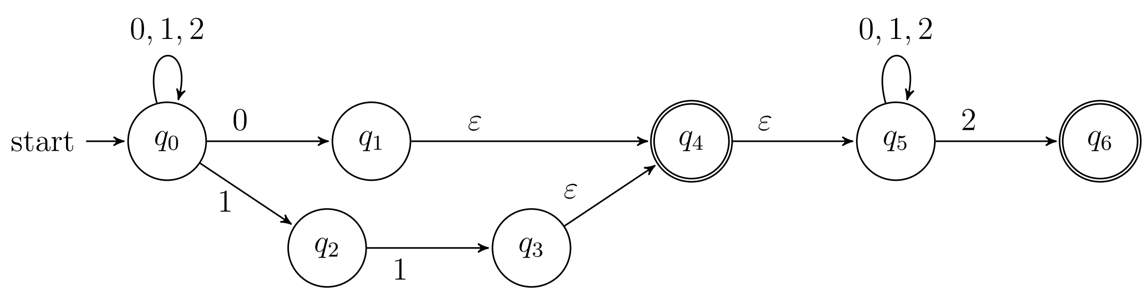
Integrity reminders

- Problems should be solved together, not divided up between the partners. The homework is designed to give you practice with the main concepts and techniques of the course, while getting to know and learn from your classmates.
- You may not collaborate on homework questions graded for correctness with anyone other than your group members. You may ask questions about the homework in office hours (of the instructor, TAs, and/or tutors) and on Piazza (as private notes viewable only to the Instructors). You *cannot* use any online resources about the course content other than the class material from this quarter – this is primarily to ensure that we all use consistent notation and definitions (aligned with the textbook) and also to protect the learning experience you will have when the 'aha' moments of solving the problem authentically happen.
- Do not share written solutions or partial solutions for homework with other students in the class who are not in your group. Doing so would dilute their learning experience and detract from their success in the class.

You will submit this assignment via Gradescope (https://www.gradescope.com) in the assignment called "hw3CSE105F24".

Assigned questions

1. **Using general constructions** (16 points): Consider the NFA N over $\{0, 1, 2\}$ with state diagram



(a) (*Graded for completeness*) [1] Give two examples of strings of length greater than 2 that are accepted by N and two examples of strings of length greater than 2 that are rejected by N.

[1]This means you will get full credit so long as your submission demonstrates honest effort to answer the

For each example string, list at least one of the computations of N on this string and label whether this computation witnesses that the string is accepted by N.

(b) (*Graded for correctness*) [2] Use the "macro-state" construction from Theorem 1.39 and class to create the DFA M recognizing the same language as N. You only need to include states that are reachable from the start state. For full credit, submit (1) a state diagram that is deterministic (there should be arrows labelled 0, 1, and 2 coming out of each state) and where each state is labelled by a subset of the states in N; and (2) for one of your example strings that is accepted by N, give the computation of M on this string as a seqeuence of states visited; and (3) for one of your example strings that is rejected by N, give the computation of M on this string as a seqeuence of states visited.

(c) (*Graded for completeness*) Give a mathematical description either using set builder notation or a regular expression for $L(N)$ and for $L(M)$.

2. **Multiple representations** (12 points):

(a) Consider the language $A_1 = \{uw \mid u \text{ and } w \text{ are strings over } \{0,1\} \text{ and have the same length}\}$ and the following argument.

> "Proof" that A_1 is not regular using the Pumping Lemma: Let p be an arbitrary positive integer. We will show that p is not a pumping length for A_1.
> Choose s to be the string 1^p0^p, which is in A_1 because we can choose $u = 1^p$ and $w = 0^p$ which each have length p. Since s is in A_1 and has length greater than or equal to p, if p were to be a pumping length for A_1, s ought to be pump'able. That is, there should be a way of dividing s into parts x, y, z where $s = xyz$, $|y| > 0$, $|xy| \leq p$, and for each $i \geq 0$, $xy^iz \in A_1$. Suppose x, y, z are such that $s = xyz$, $|y| > 0$ and $|xy| \leq p$. Since the first p letters of s are all 1 and $|xy| \leq p$, we know that x and y are made up of all 1s. If we let $i = 2$, we get a string xy^iz that is not in A_1 because repeating y twice adds 1s to u but not to w, and strings in A_1 are required to have u and w be the same length. Thus, s is not pumpable (even though it should have been if p were to be a pumping length) and so p is not a pumping length for A_1. Since p was arbitrary, we have demonstrated that A_1 has no pumping length. By the Pumping Lemma, this implies that A_1 is nonregular.

i. (*Graded for completeness*) Find the (first and/or most significant) logical error in the "proof" above and describe why it's wrong.

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question. You will not be penalized for incorrect answers. To demonstrate your honest effort in answering the question, we expect you to include your attempt to answer *each* part of the question. If you get stuck with your attempt, you can still demonstrate your effort by explaining where you got stuck and what you did to try to get unstuck.

[2]This means your solution will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

ii. (*Graded for completeness*) Prove that the set A_1 is actually regular (by finding a regular expression that describes it or a DFA/NFA that recognizes it, and justifying why) **or** fix the proof so that it is logically sound.

(b) Consider the language $A_2 = \{u1w \mid u \text{ and } w \text{ are strings over } \{0,1\} \text{ and have the same length}\}$ and the following argument.

"Proof" that A_2 is not regular using the Pumping Lemma: Let p be an arbitrary positive integer. We will show that p is not a pumping length for A_2.
Choose s to be the string $1^{p+1}0^p$, which is in A_2 because we can choose $u = 1^p$ and $w = 0^p$ which each have length p. Since s is in A_2 and has length greater than or equal to p, if p were to be a pumping length for A_2, s ought to be pump'able. That is, there should be a way of dividing s into parts x, y, z where $s = xyz$, $|y| > 0$, $|xy| \leq p$, and for each $i \geq 0$, $xy^iz \in A_2$. When $x = \varepsilon$ and $y = 1^{p+1}$ and $z = 0^p$, we have satisfied that $s = xyz$, $|y| > 0$ (because p is positive) and $|xy| \leq p$. If we let $i = 0$, we get the string $xy^iz = 0^p$ that is not in A_2 because its middle symbol is a 0, not a 1. Thus, s is not pumpable (even though it should have been if p were to be a pumping length) and so p is not a pumping length for A_2. Since p was arbitrary, we have demonstrated that A_2 has no pumping length. By the Pumping Lemma, this implies that A_2 is nonregular.

i. (*Graded for completeness*) Find the (first and/or most significant) logical error in the "proof" above and describe why it's wrong.

ii. (*Graded for completeness*) Prove that the set A_2 is actually regular (by finding a regular expression that describes it or a DFA/NFA that recognizes it, and justifying why) **or** fix the proof so that it is logically sound.

3. **Pumping** (10 points):

(a) (*Graded for correctness*) Give an example of a language over the alphabet $\{a, b\}$ that has cardinality 5 and for which 4 is a pumping length and 3 is not a pumping length. Is this language regular? A complete solution will give (1) a clear and precise description of the language, (2) a justification for why 4 is a pumping length, (3) a justification for why 3 is not a pumping length, (4) a correct and justified answer to whether the language is regular.

(b) (*Graded for completeness*) In class and in the reading so far, we've seen the following examples of nonregular sets:

$\{0^n1^n \mid n \geq 0\}$	$\{0^n1^m \mid 0 \leq m \leq n\}$	$\{0^n1^m0^n \mid n, m \geq 0\}$
$\{0^n1^n \mid n \geq 2\}$	$\{0^i1^{2i} \mid 0 \leq i\}$	$\{w \in \{0,1\}^* \mid w = w^R\}$
$\{0^n1^m \mid 0 \leq n \leq m\}$	$\{0^i1^{i+1} \mid 0 \leq i\}$	$\{ww^R \mid w \in \{0,1\}^*\}$

Modify one of these sets in some way and use the Pumping Lemma to prove that the resulting set is still nonregular.

4. **Regular and nonregular languages** (12 points): In Week 2's review quiz, we saw the definition that a set X is said to be **closed under an operation** if, for any elements in X, applying to them gives an element in X. For example, the set of integers is closed under multiplication because if we take any two integers, their product is also an integer .

Prove or disprove each closure claim statement below about the class of regular languages and the class of nonregular languages. Your arguments may refer to theorems proved in the textbook and class, and if they do, should include specific page numbers and references (i.e. write out the claim that was proved in the book and/or class).

Recall the definitions we have:

For language L over the alphabet $\Sigma_1 = \{0, 1\}$, we have the associated sets of strings

$$SUBSTRING(L) = \{w \in \Sigma_1^* \mid \text{there exist } a, b \in \Sigma_1^* \text{ such that } awb \in L\}$$

and

$$EXTEND(L) = \{w \in \Sigma_1^* \mid w = uv \text{ for some strings } u \in L \text{ and } v \in \Sigma_1^*\}$$

(a) (*Graded for completeness*) The set of regular languages over $\{0, 1\}$ is closed under the $SUBSTRING$ operation.

(b) (*Graded for completeness*) The set of nonregular languages over $\{0, 1\}$ is closed under the $SUBSTRING$ operation.

(c) (*Graded for correctness*) The set of regular languages over $\{0, 1\}$ is closed under the $EXTEND$ operation.

(d) (*Graded for correctness*) The set of nonregular languages over $\{0, 1\}$ is closed under the $EXTEND$ operation.