Regular Expressions

CSE 105 Week 1 Discussion

Deadlines and Logistics

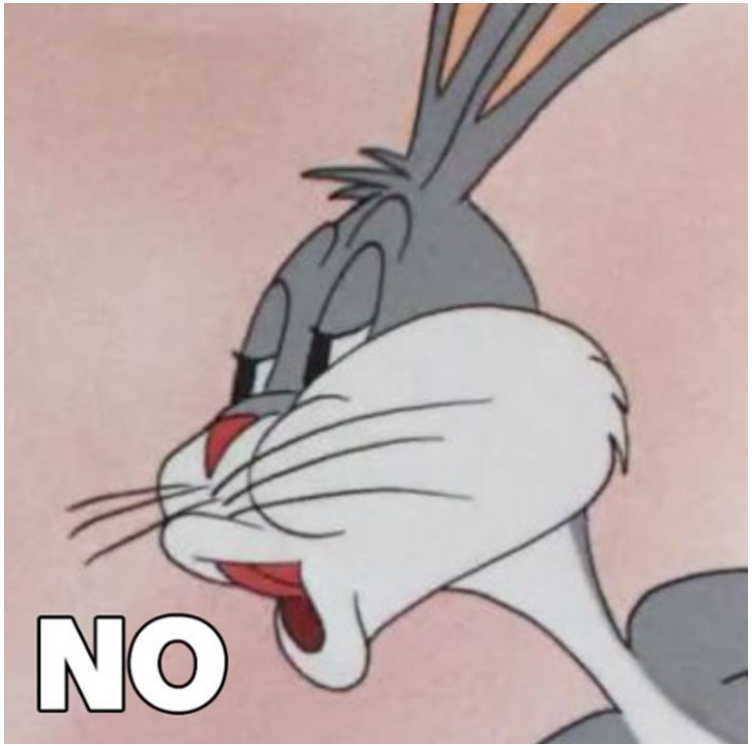
- Make sure you have access to the class website, canvas, piazza and gradescope
- Make sure you can see all our OH in the class calendar
- Make sure you have access to PrairieLearn and PrairieTest.
- Read the grading scheme on the class website !
- Schedule your tests asap on PrairieTest !
- HW1 due next week on 16th (Thursday) at 5 PM

What can you expect from the discussion section

- Recap of key concepts from lectures
- Lots of informal interaction
- Practice problems from notes, review quizzes, Sipser and HW
- Ask us any questions and get your doubts answered !

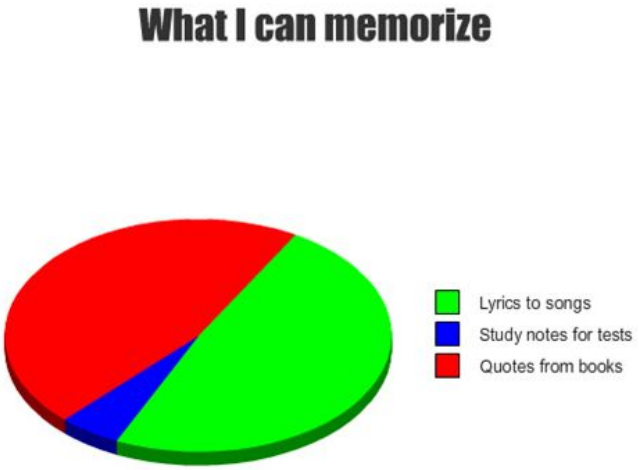
PS - All screenshots taken either from Sipser or class notes unless specified otherwise

Are the discussion sections podcasted/online/recorded ?



General Information

Do not memorize anything for the sake of completion of HWs, exams etc without first understanding underlying concepts !



Motivation

1. What are the capabilities and limitations of computers ?
2. Can we answer how “difficult” (or if even possible) a certain computation task is ?
3. Can we mathematically model computational problems ?

Important notation

Alphabet	A non-empty finite set, usually denoted as Σ
Symbol	An element of the alphabet
String over Σ	A finite list of symbols from Σ
Language over Σ	A set of strings over Σ
Σ*	Set of all possible strings formed from symbols in Σ

Is this statement correct?

Given Language L = {w|w is a string over {0} such that |w| = 1 }, then L = 0.

Review Quiz Question

Consider the language $\{w \mid w \text{ is a string over } \{0,1\} \text{ and } |w| \text{ is an integer multiple of } 3\}$. Which of the following are elements of this language? (Select all and only that apply)

- [ ] 000
- [ ] 0
- [ ] The empty set
- [ ] $(1,0,1)$
- [ ] $\{000\}$
- [ ] The empty string

Select all possible options that apply.

What option(s) can you eliminate right away by virtue of the definition of strings and languages ?

Review Quiz Question

Consider the language $\{w \mid w \text{ is a string over } \{0,1\} \text{ and } |w| \text{ is an integer multiple of } 3\}$. Which of the following are elements of this language? (Select all and only that apply)

- [ ] 000
- [ ] 0
- [ ] [redacted]
- [ ] [redacted]
- [ ] [redacted]
- [ ] The empty string

Select all possible options that apply.

Can eliminate all non string options ! (since only strings can be elements of a language

What option(s) can you eliminate right away by virtue of the definition of strings and languages ?

Regular Expressions

Remember that at the end of the day, Languages are sets. How can we define a set?

1. List out all the elements
2. Use set builder notation and describe membership condition

Or

3. Use recursive definitions (Regular Expressions !)

The language described by a regular expression *R* is *L(R)*

Recap

Let A and B be languages. We define the regular operations ***union***, ***concatenation***, and ***star*** as follows:

- **Union**: $A \cup B = \{x | \, x \in A \text{ or } x \in B\}$.
- **Concatenation**: $A \circ B = \{xy | \, x \in A \text{ and } y \in B\}$.
- **Star**: $A^* = \{x_1 x_2 \ldots x_k | \, k \geq 0 \text{ and each } x_i \in A\}$.

Pop quiz : If ∑ = {0,1} and language A = {0, 01, 1} and B = {ε, 1} What is

- A ∪ B
- A○B
- B*
- Is (A*)* = A* ?

Recap

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Pop quiz : If Σ = {0,1} and language A = {0, 01, 1} and B = {ε, 1} What is

- A ∪ B : {ε, 0, 1, 01}
- A∘B : {0, 01, 1, 011, 11}
- B* : {ε, 1, 11, 111 ....} or $\{1_k | \text{ k is a non negative integer}\}$
- Is (A*)* = A* ? : Yes ! Both expressions produce the set of all strings that can be formed by concatenating strings in A with one another as many times as we want

R is a regular expression over the alphabet Σ

1. $R = a$, where $a \in \Sigma$
2. $R = \varepsilon$
3. $R = \emptyset$
4. $R = (R_1 \cup R_2)$, where R_1, R_2 are themselves regular expressions
5. $R = (R_1 \circ R_2)$, where R_1, R_2 are themselves regular expressions
6. (R_1^*), where R_1 is a regular expression.

Context is super important !

For the following examples assume the alphabet is $\Sigma_1 = \{0, 1\}$:

The language described by the regular expression 0 is $L(0) = \{0\}$

The language described by the regular expression 1 is $L(1) = \{1\}$

The language described by the regular expression ε is $L(\varepsilon) = \{\varepsilon\}$

The language described by the regular expression $\emptyset$ is $L(\emptyset) = \emptyset$

The language described by the regular expression $(\Sigma_1\Sigma_1\Sigma_1)^*$ is $L(\ (\Sigma_1\Sigma_1\Sigma_1)^*\ ) =$

Do both the Σ_1 refer to the same thing ?

Context is super important !

Refers to the alphabet set Σ_1 containing symbols 1 and 0 !

For the following examples assume the alphabet is $\Sigma_1 = \{0, 1\}$:

The language described by the regular expression 0 is $L(0) = \{0\}$

The language described by the regular expression 1 is $L(1) = \{1\}$

The language described by the regular expression ε is $L(\varepsilon) = \{\varepsilon\}$

The language described by the regular expression $\emptyset$ is $L(\emptyset) = \emptyset$

The language described by the regular expression $(\Sigma_1\Sigma_1\Sigma_1)^*$ is $L(\ (\Sigma_1\Sigma_1\Sigma_1)^*\ ) =$

Do both the Σ_1 refer to the same thing ? : NO !

Refers to one occurrence of any symbol (0 or 1 in this case) from Σ_1.

Other conventions

Assuming Σ is the alphabet, we use the following conventions

Σ	regular expression describing language consisting of all strings of length 1 over Σ
$*$ then $\circ$ then $\cup$	precedence order, unless parentheses are used to change it
R_1R_2	shorthand for $R_1 \circ R_2$ (concatenation symbol is implicit)
R^+	shorthand for $R^* \circ R$
R^k	shorthand for R concatenated with itself k times, where k is a (specific) natural number

Remember

A regular expression should describe a SET of strings over an alphabet since it is descriptive of a Language, which is a SET of strings over an alphabet.

All “operations” and “conventions” you see in a regular expression boil down to some fundamental operation(s) on set(s).

What operations and what sets ?

Remember

A regular expression should describe a SET of strings over an alphabet since it is descriptive of a Language, which is a SET of strings over an alphabet.

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What operations and what sets ?

U, * and ○

As a basis step :

- Element ∈ alphabet or
- ε
- Ø

Inductively:
On any regular expression(s)

When you see... The language described by the regular expression $1^* \circ 1$ is $L(1^* \circ 1) =$

You need to think : sets and operations !

Solve :

When you see... The language described by the regular expression $1^* \circ 1$ is $L(1^* \circ 1) =$

You need to think : sets and operations !

Solve :

$L(1^*\circ1)$
$= L(1^*) \circ L(1)$
$= L(1)^* \circ L(1)$
$= \{1\}^* \circ \{1\}$
$= \{1^k \mid k>=0\} \circ \{1\}$
$= \{1^k1 \mid k>=0\}$ (sufficient to leave it at this step)
$= \{1^k \mid k>=1\}$

From Regex to Language

The language over $\Sigma_1 = \{0, 1\}$ described by the regular expression Σ_1^*1 is $L(\Sigma_1^*1) =$ BLANK

Describe this set in (a) Simple english and (b) Set builder notation

(a) Set of all strings over Σ_1 ending in a 1

(b) $\{x1 \mid x \in \Sigma_1^*\}$

Describe the language generated by these Regex (over ∑ = {0,1}) : Basics

1. (0U1)
2. (0U1)*
3. ∑*
4. (0)U(1)
5. (01) U (1)
6. (01)* U (1)
7. (01 U 1)*

Describe the language generated by these Regex(over $\Sigma = \{0,1\}$) : Basics

1. (0U1) : $\{0,1\}$
2. (0U1)* : $\{x \mid x \in \Sigma^*\}$
3. Σ^* : Set of all strings that can be created from elements in Σ (Notice similarity to (2)
4. (0)U(1) : $\{0, 1\}$
5. (01) U (1) : $\{01, 1\}$
6. (01)* U (1) : $\{(01)^k \mid k>=0\} \cup \{1\}$
7. (01 U 1)* : $\{x \mid x \in \{01,1\}^*\}$

Describe the language generated by these Regex(over ∑ = {0,1}) : Level up

1. (01)(01)*(01)
2. $(01)^{+}(01)$
3. 1 U 11 U 111 U 1111 U 11111*
4. ε*∑∑∑ε*
5. (ε U ∑)(ε U ∑)(ε U ∑)
6. ∑*1∑*0∑* U ∑*0∑*1∑*
7. (1∑*1) U (0∑*0) U (ε) U (1) U (0)

Describe the language generated by these Regex(over ∑ = {0,1}) : Level up

1. (01)(01)*(01) : Set of strings containing repeating units of (01) with at least 2 repeats
2. $(01)^{+}(01)$: Set of strings containing repeating units of (01) with at least 2 repeats
3. 1 U 11 U 111 U 1111 U 11111* : Same language as 11* or 1^{+}
4. ε*∑∑∑ε* : All strings of length 3
5. (ε U ∑)(ε U ∑)(ε U ∑) : All strings of length at most 3
6. ∑*1∑*0∑* U ∑*0∑*1∑* : All strings containing at least one 1 and one 0
7. (1∑*1) U (0∑*0) U (ε) U (1) U (0) : All strings starting and ending with the same symbol

Questions?

Good luck for HW 1 !