\[ \Sigma = \{a, b, c\} \]

**String:** sequence of symbols

Examples of strings over \( \Sigma \):
- abc
- a
- \( \varepsilon \)
- cccccc

A string is **over** \( \Sigma \) if all of its symbols are in \( \Sigma \)

A **language** over \( \Sigma \) is a set of strings over \( \Sigma \)

Examples of languages over \( \Sigma \):
- \{a, b, c\}
- \{abc, a, \( \varepsilon \), b, cccccc\}
- \{b^n \mid n \geq 0\}
- \{\varepsilon\}
- \{b^n \mid n \geq 0\}

We can use regular expressions to describe these languages.
Regular Expressions

Basis steps:
- $a$ is a regular expression, for $a \in \Sigma$
- $\epsilon$ is a regular expression
- $\emptyset$ is a regular expression

Recursive steps:
- $(R_1^*)$ is a regular expression when $R_1$ is a regular expression
- $(R_1 \circ R_2)$ is a regular expression when $R_1$ and $R_2$ are regular expressions
- $(R_1 \cup R_2)$ is a regular expression when $R_1$ and $R_2$ are regular expressions
Regular Expressions

$\Sigma = \{a, b, c\}$

$(R_1^*)$ is a regular expression when $R_1$ is a regular expression

$b^*$

$L(b) = \{b\}$

$L(b^*) = \{b^n \mid n \geq 0\}$

$\Sigma^* L(\Sigma^*) = \{b^n \mid n \geq 0\}$

the set of strings over $\Sigma$

$(R_1 \circ R_2)$ is a regular expression when $R_1$ and $R_2$ are regular expressions

$a \in a \in a \in a \in a$

$L(aaaaa) = \{aaaaa\}$

$C \in b^*$

$L(Cb^*) = \{cb^n \mid n \geq 0\}$

$c \in cb \in cb \in cb \in cb$

$(R_1 \cup R_2)$ is a regular expression when $R_1$ and $R_2$ are regular expressions

$a \cup c = L(a) \cup L(c) = \{a, c\}$

$Cb^* \cup aaaaa = L(cb^* \cup aaaaa) = L(cb^*) \cup L(aaaaa) = \{cb^n \mid n \geq 0\} \cup \{aaaaa\}$
\[ \Sigma = \{a, b, c\} \]

**Regular Expressions**

\[ cb^* u aaaaa \]

L(\(cb^* u aaaaa\)) = \{\(cb^n\) | \(n \geq 0\)\} \(\cup\) \{aaaaa\}

\[ cb\cb\cb\cb\cb \ aaaaa \]

Implicitly:

\((c(b^*)) \cup (aaaaa)\)

**What happens when we evaluate in a different order?**

\[ (Cb)^* u (aaaaa) \]

L(\((Cb)^*\)) = \{\(Cb^n\) | \(n \geq 0\)\}

\[ cbcbcbcb \ \in \ \Sigma \]

Examples of strings

The language is not the same!

Precedence order: First \(*\), then \(\circ\), then \(\cup\)