DISCUSSION 04/12

For definitions, refer Sipser.

\( \Sigma = \{a, b\} \)

14 (a) \{w \mid \text{w has at least 3 'a's and at least 2 'b's}\}

**Sol:** We want the intersection of 2 DFAs \( M_1 \) & \( M_2 \)

- \( M_1 \) recognizes \( \{w \mid \text{w has at least 3 'a's}\} \)
- \( M_2 \) recognizes \( \{w \mid \text{w has at least 2 'b's}\} \)

![Diagram of DFAs M1 and M2](image)

**M1**

\[
\begin{align*}
& \overset{b}{\rightarrow} q_1 \\
& \overset{a}{\rightarrow} q_2 \\
& \overset{b}{\rightarrow} q_3 \\
\end{align*}
\]

**M2**

\[
\begin{align*}
& \overset{a}{\rightarrow} r_1 \\
& \overset{b}{\rightarrow} r_2 \\
& \overset{a}{\rightarrow} r_3 \\
\end{align*}
\]

\( M_1 \cap M_2 \)
1.4 (c) \{ w \mid w \text{ has an even number of } a \text{'s and } 1 \text{ or } 2 \text{ 'b's} \}

\[ \text{Sol: Want } M_1 \cap M_2 \text{, where} \]

\[ M_1 \text{ recognizes } \{ w \mid w \text{ has an even number of } a \text{'s} \} \]

\[ M_2 \text{ recognizes } \{ w \mid w \text{ has } 1 \text{ or } 2 \text{ 'b's} \} \]

\[ M_1 \cap M_2 \]

1.5 (c) Construct a DFA that recognizes

\[ L = \{ w \mid w \text{ contains neither } ab \text{ nor } ba \} \]

The complement of this language is

\[ \overline{L} = \{ w \mid w \text{ contains } ab \text{ or contains } ba \} \]
DFA that recognizes $\overline{L}$:

![DFA Diagram]

Complement this to get the DFA that recognizes $L$:

![Complemented DFA Diagram]

15(e) $\{ w | w \text{ is any string not in } (ab^+)^* \} = \overline{L}$

$\overline{L} = \{ w | w \text{ is in } (ab^+)^* \}$

![Complemented DFA Diagram]

recognizes $\overline{L}$

Complement this:

![Complemented DFA Diagram]

recognizes $L$
1.6 (i) Construct a DFA that recognizes
\( \{ w | \) every odd position of \( w \) is a 1 \} 
over \( \Sigma = \{0, 1\} \)

Solution: \( \epsilon \) (empty string) BELONGs to this language.